Multiobjective Optimal Placement of Convectively Cooled Electronic Components on Printed Wiring Boards

Nestor V. Queipo, Joseph A. C. Humphrey, and Alfonso Ortega, Member, IEEE

Abstract—This paper presents a solution methodology for multiobjective optimization problems in the context of models for the placement of components on printed wiring boards (PWB's). The methodology combines the use of a flow and heat transfer solver, a genetic algorithm for the adaptive search of optimal or near-optimal solutions, and a multiobjective optimization strategy (Pareto optimization or multiattribute utility analysis (MUA)). Using as the optimization criterion the minimization of an estimate of the failure rate of the system of components due to thermal overheating (via an Arrhenius relation), the effectiveness of the present solution methodology is demonstrated by reference to a case with known optimal solutions. The results obtained using the same solution methodology for a multiobjective optimization problem (a variation of the case study) involving the minimization of the aforementioned total failure rate of the system as well as the minimization of the total wiring length (given some interconnectivity requirements) are presented and discussed for both Pareto optimization and MUA.

Index Terms—Electronics cooling, genetic algorithms, multiobjective optimal placement, printed wiring boards, thermal management.

I. INTRODUCTION

The optimal placement of components on printed wiring boards requires satisfying multiple, possibly conflicting, design objectives. As pointed out by Moresco [1], these design objectives may be very different in nature—geometrical, electrical, thermal, mechanical, and cost (manufacturing and maintenance)—which makes finding the “best” design a complicated task.

Two major design objectives are related to the reliability (thermal/mechanical/cost) and the routing (electrical/cost) requirements of the component placement design. Specifically, the minimization of estimates of the failure rate of the system and total wiring length are design objectives of prominent interest. The former optimization criterion imposes major heat transfer requirements on the design because of the combined effects of:

1) rapidly increasing packaging density and power dissipation demands;
2) potentially high costs associated with the failure of electronic components, as pointed out by Weiss et al. [2] and Wessely et al. [3] among others.

The latter is critical because of electrical performance, speed, and transmission line requirements and its impact on the manufacturing costs.

Most optimization studies regarding component placement have considered a single design objective, such as reliability (see, for example, [4]–[6]), or routing (see, for example, [7], [8], and many others reported in [9]). With the exception of the work by Osterman et al. [10], the few optimization studies that have addressed multiple objectives have failed to provide rigorous methods to select the “best” design [11] or have made somewhat arbitrary choices regarding the relative importance of the design objectives under consideration [12]–[14]. The solution methodology proposed by Osterman et al. [10] includes a thermal resistive network approach for predicting the temperatures in the heated elements, an optimization strategy based on the force-directed placement methodology and the weighting method for combining the various optimization objectives.

This study overcomes some of the limitations of previous studies in this area and discusses a methodology to select the “best” component placement design when multiple design objectives are present. The methodology is based on the concepts of Pareto optimality [15] and multiattribute utility analysis (MUA) [16]. The Pareto optimization provides a set of alternative component placements from which the “best” design must be selected, and the MUA assists in the process of articulating the designer’s preferences and identifying the “best” component placement (decision problem). As reported by Thurston [17], the MUA has been successfully applied to a wide variety of decision problems, including trajectory selection for NASA missions, nuclear power plant site selection, telecommunication system architecture design, and many others.

The remainder of this paper is structured as follows. Section II provides a formal definition of the problem of interest and Section III gives a description of the different elements of the present solution methodology and their...
interaction. In particular, Section III describes a flow and heat transfer solver, a genetic algorithm and two different multiobjective optimization strategies (Pareto optimization and MUA). A description of a case study designed to validate and evaluate the present solution methodology is the subject of Section IV. The paper ends with the application of the present solution methodology to a multiobjective optimization problem (a variation of the case study) using both Pareto optimization and MUA.

II. PROBLEM DEFINITION

The problem of interest here corresponds to the optimal placement of convectively cooled electronic components on printed wiring boards (PWB) subject to thermal and nonthermal optimization criteria. Because of its cost effectiveness and mechanical simplicity, forced air cooling is the most frequently used technique for cooling electronic components in personal computers and workstations. These systems comprise a major portion of the market with moderate heat transfer rate requirements. The convectively cooled electronic components on printed wiring boards are modeled here as equally spaced heated elements placed on the bottom wall of a ventilated two-dimensional (2-D) channel, as illustrated in Fig. 1. The printed wiring board is aligned parallel to the coolant flow which is assumed laminar and 2-D. Each component is assumed to dissipate a constant heat flux and the heat fluxes may differ among components.

Regarding thermal optimization, forced air cooling is usually limited by acoustic noise constraints placed on the fan driving the flow, and arrangements of electronic components that maximize reliability and minimize thermo-mechanically induced stresses are highly desirable. Examples of nonthermal optimization criteria include the need to minimize the total wire length on the PWB, clustering functionally related components to conform to speed and transmission line requirements, and keeping analog components and digital components separate to reduce crosstalk.

In this study, the minimization of the failure rate of the electronic components on the printed wiring boards due to thermal overheating, and the minimization of the total wiring length satisfying the requirements specified by an interconnectivity matrix, are selected as thermal and nonthermal optimization criteria, respectively.

The reliability prediction is the statistical estimate of the value of time over which a device will function. The inverse of the reliability of a device is called its failure rate and is measured in failures per megahours (fr Mh$^{-1}$). As indicated by, for example, Moresco et al. [1], and Wessely et al. [3], the failure rate of an electronic component is a strong function of its temperature.

Even though various functional relationships between failure rate and temperature in electronic components have been suggested (Wong [18]), according to Blanks [19], the Arrhenius relation is the most widespread model among practitioners in the electronic packaging industry. In this study, the failure rate of electronic component "i" is estimated using the Arrhenius relation as

$$\lambda_i = A_i \exp\left(-\frac{B_i}{T_{i}^{\text{max}}}ight).$$

Here $A_i$ and $B_i$ are constants associated with the thermal sensitivity of the electronic component, while $T_{i}^{\text{max}}$ is the maximum temperature of component "i." Of interest here is the general case for which the electronic components on the PWB may differ in heat dissipation rate and thermal sensitivity. Since the failure rate of a component depends strongly on temperature as specified by (1), the maximum temperature of each component is calculated by solving numerically the conservation equations for continuity, momentum, and energy.

One of the objective functions to be minimized in this study is the total failure rate of a system consisting of a number of electronic components equal to $N_{\text{com}}$ and given by the sum of the individual component failure rates as shown in

$$\lambda_{\text{total}} = \sum_{i}^{N_{\text{com}}} \lambda_i.$$

The wiring requirements among different components is represented by an interconnectivity matrix ($J$). An entry $I_{i,j}$ in the interconnectivity matrix (see Fig. 2) is given the value 1 if component "i" is functionally related to component "j" or the value 0 otherwise.

If we denote the wiring length between components "i" and "j" by the variable $L_{i,j},$ the additional objective function to be
satisfied is the minimization of
\[
g = \sum_{i=1}^{N_{\text{com}}} \sum_{j=1}^{i} L_{ij} \cdot I_{ij}.
\] (3)

In summary, the problem of interest may be stated as follows: given \(N_{\text{com}}\) heated elements to be distributed among \(N_{\text{com}}\) equally-spaced locations on the bottom wall of a 2-D ventilated channel, what are some of the arrangements that minimize both a measure of the failure rate of the system and the total wiring length required to meet the wiring requirements associated with a given interconnectivity matrix?

III. SOLUTION METHODOLOGY

The solution methodology, illustrated in Fig. 3, has three elements: a flow and heat transfer solver, a genetic algorithm, and a multiobjective optimization strategy. The flow and heat transfer solver is responsible for the accurate prediction of the maximum temperature of each heated element used for calculating the individual failure rates. The multiobjective optimization strategy provides the means to convert the original multiobjective optimization problem into a form amenable to be solved by the genetic algorithm. The genetic algorithm is responsible for the adaptive search of optimal or near-optimal solutions. Note that even for the simplified model formulated in this study the thermal optimal placement of electronic components with different heat generation rates and thermal sensitivities would require an exhaustive investigation of the entire solution space which, in this case, is combinatorial. For example, if eight different components are considered, the number of possible arrangements is 40,320 (8!) and, as indicated by De Jong [20], nonadaptive search procedures may be computationally prohibitive. Specifically, for the case of interest, the time needed to compute the maximum heated element temperatures for any given arrangement (see next section) on a workstation (IBM RISC 6000/530) was approximately 30 min, and as result, it would take about 2.3 years to exhaustively investigate the solution space.

A. Flow and Heat Transfer Solver

For the purpose of estimating the failure rate of a given arrangement of electronic components on a printed wiring board using the model adopted in this investigation, it is necessary to estimate the maximum temperature on the surface of each heated element. This temperature is a function of the air velocity field, the thermal boundary conditions exhibited by the heated elements, the specific geometry (size and height of the heated elements), distance between heated elements, substrate conduction characteristics, and the distance from the inflow boundary to the first heated element. If the substrate is assumed to be adiabatic, and the heat fluxes on the element surfaces are specified, the maximum surface temperature of each heated element can be obtained by solving the conservation equations of mass, momentum and energy in the fluid phase are given by (4)–(7), respectively:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\] (4)
\[
\frac{\partial U U}{\partial X} + \frac{\partial V}{\partial Y} - \frac{\partial P}{\partial X} + \frac{1}{Re} \left\{ \frac{\partial U}{\partial X^2} + \frac{\partial U}{\partial Y^2} \right\}
\] (5)
\[
\frac{\partial U V}{\partial X} + \frac{\partial V}{\partial Y} - \frac{\partial P}{\partial Y} + \frac{1}{Re} \left\{ \frac{\partial V}{\partial X^2} + \frac{\partial V}{\partial Y^2} \right\}
\] (6)
\[
\frac{\partial U \theta}{\partial X} + \frac{\partial V \theta}{\partial Y} = \frac{1}{Pr Re} \left\{ \frac{\partial \theta}{\partial X^2} + \frac{\partial \theta}{\partial Y^2} \right\}
\] (7)

The dimensionless quantities appearing in these equations are

\[
X = \frac{x}{h}, \quad Y = \frac{y}{h}, \quad U = \frac{u}{u_0}, \quad V = \frac{v}{u_0}, \quad \theta = \frac{T - T_0}{q_a h / k}, \quad Re = \frac{u h}{v}, \quad Pr = \frac{\nu}{\alpha}.
\]
In these expressions, the symbols $\alpha$ and $\nu$ denote thermal diffusivity and kinematic viscosity, respectively. The symbol $q^i_w$ represents the total heat flux provided by any one of the heated elements taken as a reference value.

For the channel flow considered in this investigation (see Fig. 1) uniform velocity and temperature distributions are imposed at the inlet. Except for the inflow boundary, all exposed channel surfaces are taken as adiabatic. In general, selecting accurate boundary conditions at open boundaries has been shown to be difficult [22]. In this study, the streamwise velocity and temperature gradients are assumed to be zero at the outlet. Blosh et al. [23] have shown that the specification of velocity gradients equal to zero is a good boundary condition for the open boundary of channel flow configurations, provided the velocity at the outlet is corrected so that global conservation of mass is enforced. The temperature gradient equal to zero (thermally fully developed flow) is imposed at a distance sufficiently removed from the last heated element so that the computed maximum temperature of each heated element is unaffected.

The boundary conditions are as follows:

- **Inlet plane**: $U = 1.0$, $V = 0$, $\theta = 0$;
- **Outlet plane**: $\partial U / \partial X = 0$, $\partial V / \partial X = 0$, $\partial \theta / \partial X = 0$, and $\int_{U_{out}} U dY = 1$;
- **Top and bottom walls**: $U = V = 0$;
- **Heated elements**: $U = V = 0$, $\partial \theta / \partial Y = 0$;
- **Heated elements**: $U = V = 0$, $\partial \theta / \partial Y|_{last} = q^i_q / q^i_w$.

The initial condition imposed on the flow field calculations corresponded to developed flow in a channel at every streamwise location except within the solid heated elements where velocities were set to zero. The initial temperature field in the fluid was set equal to the inlet temperature.

The configuration geometry is specified by the number of heated elements, the channel height ($h$), the heated element width ($w$), heated elements height ($h_1$, $h_2$), the inter-element spacing ($s$), the distance from the inlet plane to the first element ($l_1$) and the distance from the last element to the exit plane ($l_2$) (see Fig. 1).

The nondimensional values adopted for these quantities are summarized in Table I and are close to the values reported by Kim [24] as typical of models of electronic components on printed wiring boards. The value of the geometrical parameter $l_2/h$ is selected such that the location of the exit plane does not significantly affect the maximum temperatures calculated on the surfaces of the heated elements. As in Queipo et al. [12], the number of heated elements considered is eight.

In this study, the cooling fluid is air and all physical properties are evaluated for air at 300 K. The channel height was assumed to be 0.02 m and the inlet velocity $u_0 = 0.59$ m/s, corresponding to a Reynolds number of $Re = 750$. The Prandtl number of air at the reference temperature was taken as $Pr = 0.7$.

The program flow and heat transfer solver (FHTS) was used to perform the numerical calculations of flow and heat transfer. The program finds its origins in the ROTFLO2 program developed by Hayase et al. [25] and allows the direct numerical simulation of unsteady, three-dimensional, nonisothermal, constant property laminar flow in Cartesian or cylindrical coordinates. The numerical procedure solves for the primitive variables (velocity and pressure) and is based on the finite difference equations derived using the staggered grid control-volume formulation presented by Patankar [26], but with the convective coefficients discretized using the QUICK scheme as suggested by Hayase et al. [27]. FHTS includes the codification of a variety of velocity-pressure coupling algorithms such as the SIMPLE procedure of Patankar and Spalding [28], the SIMPLER procedure of Patankar [26], and the SIMPLE procedure of VanDoornmaal et al. [29]. The program FHTS has been described and successfully tested in Queipo [14] using a variety of standard flow and heat transfer benchmark test cases. These include the cavity driven flow of Ghia et al. [30], and the backward facing step flow and heat transfer of Gartling [31] and Runchal [32], among others.

The velocity field for each configuration is calculated using the SIMPLE algorithm with an under-relaxation factor for the velocities $\alpha = 0.7$. The under-relaxation factor for pressure was taken as $1-\alpha$. According to Peric [34] (see Dainese [33]) this relation between the velocity and pressure under-relaxation factors is optimal. Convergence was achieved when a variable ($\epsilon$) representing the maximum mass, $U^\prime$-momentum and $V^\prime$ momentum residual fell below a given predefined value. These residuals are computed as the sum of the absolute values of the corresponding mass or momentum imbalances over each of the control volumes in the domain. A convergence criteria of $\epsilon \leq 10^{-4}$ was used for computing the velocity field.

The temperature field is obtained using an under-relaxation parameter equal to 0.9. Convergence in the numerical calculation of temperature was achieved when a variable $\epsilon$ representing the energy residual fell below a given predefined value. This residual is computed as the sum of the absolute value of the energy imbalance of each of the control volumes in the domain. A convergence criteria of $\epsilon \leq 10^{-5}$ was used to compute the temperature field.

### B. Genetic Algorithm

Genetic algorithms are adaptive search procedures loosely based on the Darwinian notion of evolution that have been employed successfully in a variety of search, optimization and machine learning applications. The genetic algorithm in this study corresponds to the Combinatorial Simple Genetic Algorithm encoded in the program CSGA, documented in Queipo [14]. The CSGA program has the structure of the program GAucsd (v. 1.4) developed by Schraudolph et al. [35], but uses a different representation (integer representation) and different recombination operators (partially matched crossover). In addition, the random number generator in the program CSGA is the routine RAN2 available in Numerical
Recipes by Press et al. [36]. For a general introduction to genetic algorithms, see [37] or [38]. An introduction to genetic algorithms in the context of thermosciences applications is given by Queipo et al. [13].

The interaction between the flow and heat transfer solver and the genetic algorithm is illustrated in Fig. 3. There are two key elements to consider in describing the connection between CSGA and FHTS:

1) the control structure of their coupled execution;
2) the information exchange between the two programs.

During the coupled execution of the CSGA and the FHTS programs, CSGA is the master process and FHTS is the slave process. Each time the program CSGA requires the evaluation of a new candidate solution, a slave process is created and the execution of CSGA is suspended. Within the slave process, the program FHTS is invoked and after its successful completion, CSGA resumes its execution. All this is done within a UNIX operating system environment.

The CSGA and the FHTS programs exchange information through data files. The program CSGA makes available to FHTS two files:

1) a file called comp.dat describing the geometrical and thermal characteristics of the heated elements in the candidate solution;
2) a file called sequence.dat describing the order in which the heated elements specified in components.dat are positioned along the bottom wall of the ventilated channel.

The program FHTS generates the file temp.dat after its successful execution. The file temp.dat contains the maximum temperature on the surface of each of the heated elements in the candidate solution.

C. Multiobjective Optimization

In contrast to the optimization of a single function where the term optimum value has a unique meaning and geometric interpretation, in the case of multiobjective optimization there is not a general definition of the optimal values. Here, the term optimization means to find a solution that provides acceptable values for the objective functions and that satisfies the preference structure of the person posing the problem; that is, the designer.

Hence, the problem in multiobjective optimization consists in finding a vector of design variables that satisfies a set of constraints and that optimizes a second vector whose elements represent the objective functions. There is no single best approach for solving these problems. Different philosophies and methodologies co-exist for addressing optimization problems with multiple objectives. The approaches differ in their view concerning whether or not it is possible (or practical) to capture the preference structure of the designer. The spectrum of methods begins with Pareto optimization where there is no information regarding the preference structure of the designer, and ends with the MUA (Keeney et al. [16]) where it is assumed possible to capture the aforementioned preference structure.

1) Pareto Optimization: A vector of decision or design variables belongs to the Pareto optimal set or set of non-dominated solutions if there is no other solution that could improve the value of one of the objective functions without deteriorating at least one of the others objective functions. Examples of Pareto solutions are the solutions obtained by optimizing the objective functions individually.

In the case of Pareto optimization, no information is assumed regarding the designer except for his “preference independence.” Preference independence describes the situation where lowering the values of the objective function is always better (assuming the problem is one of minimization). The methods in this category attempt to provide a representative approximation of the Pareto optimal set and some of the criteria to evaluate such methods include:

1) how good is the approximation provided by the method of the Pareto optimal set and if it is able to generate a nonconvex Pareto set; that is, a Pareto set represented by a nonconvex curve;
2) how fast the computational effort of its use grows with respect to the number of variables;
3) how easy it is to implement.

Some of the methods that belong to this category are: the weighting method, the noninferior set method and the restriction method (Balachandran et al. [15]).

The Pareto optimization in this work is conducted using the weighting method. The weighting method converts the multiobjective problem to a scalar optimization problem, in which the objective function becomes a weighted sum of the individual objective functions. That is

$$\min \sum_{i=1}^{n} u_i f_i(\pi) \quad 1 \leq i \leq n$$

wherein, the $u_i$'s represent the weights and the $f_i$'s represent the individual objective functions. The above problem is a single-objective optimization problem and it is solved using a genetic algorithm. This is a very simple approach that fits the purpose of this investigation. However, the weighting method is not without its drawbacks: it does not uncover solutions in nonconvex regions of the Pareto optimal set; and it finds the Pareto optimal set by solving multiple scalar optimization problems (different set of weights) which may be computationally expensive.

Studies of Pareto optimization using genetic algorithms to obtain the set of nondominated solutions at once have been attempted. The first effort in the use of genetic algorithms in multiobjective optimization problems (Pareto optimization) is due to Shaffer [39]. In his genetic algorithm the population is divided into sub-populations with the fitness of the chromosomes in different sub-populations being evaluated using the different objective functions. Shaffer’s approach has the problem that it does not provide a uniform approximation of the Pareto set with the solutions obtained concentrated around the extremes of the nondominated solutions set. A recent genetic algorithm claiming to provide a good approximation of the Pareto optimal set using genetic algorithms is reported by Horn et al. [40].

2) Multiattribute Utility Analysis: Pareto optimization is a member of a family of methods based on the measurement of
the values of each objective function and on the knowledge of their relative priority. While this approach may be found useful, as pointed out by Thurston [17], it is limited in two respects:

1) the direct measurement of the objective functions or attributes of the design, does not necessarily reflect the subsequent value or worth to the designer;

2) methods that rely on the concept of relative importance or priority might not accurately quantify attribute tradeoffs.

Attribute tradeoffs refer to the designer’s willingness to “pay” for improvement in one attribute at the expense of the other. In contrast to Pareto optimization, MUA concentrates on finding the overall value of the designs; hence, the design with the highest value to the designer can be identified.

The MUA method becomes practical when the so called preferential and utility independence assumptions are met. Preferential independence makes reference to situations where the designer always prefers less to more of an attribute (or more to less depending of the attribute) regardless of the level of the other attributes. Utility independence means that the general shape of the utility functions associated with each attribute (to be discussed later) is not altered by levels of the other attributes. Under this conditions, the overall worth of a design \( U(f) \) can be calculated using (8) (see Keeney et al. [16])

\[
U(f) = \frac{1}{K} \left( \sum_{i=1}^{n} k_i U_i(f_i) + 1 \right) - 1
\]

wherein

- \( U(f) \) overall worth of the set of attributes \( f_i \);  
- \( f_i \) level of attribute \( f_i \);  
- \( f \) set of attributes levels \( (f_1, f_2, \cdots, f_n) \);  
- \( k_i \) assessed single attribute scaling constant;  
- \( U_i(f_i) \) assessed single attribute utility function;  
- \( K \) scaling constant;  
- \( n \) number of attributes.

If the more restrictive additive independence condition reported by Thurston [17] is satisfied, that is

\[
\sum_{i=1}^{n} k_i = 1.
\]

It can be shown that (8) reduces to

\[
U(f) = \sum_{i=1}^{n} k_i U_i(f_i).
\]

Equation (10) leaves the designer with two tasks:

1) the identification of the worth of the different levels of each attribute in isolation expressed in the single attribute utility function \( U_i(f_i) \);

2) a measure of the tradeoffs the designer is willing to make, in the form of the attribute’s scaling constant \( k_i \).

The constants \( k_i \) should not be confused with relative importance of attributes or weighting factors.

Points in the single attribute utility functions \( U_i(f_i) \) and the attribute’s scaling constant \( k_i \) can be obtained using the “certainty equivalent” method. An example of the lottery questions used in the certainty equivalent method to assess the single attribute utility function \( U_i(f_i) \) for failure rate.

The designer is asked to imagine two alternative designs: the “certain” alternative is known with certainty to be some value \( f_i \), while the “lottery” alternative represents a design alternative in which there is uncertainty as to the attribute level. The lottery in Fig. 4, shows a probability \( p \) of 30% that the failure rate \( f_i \) will be at the estimated best possible level \( f_{ib} \) and a probability of \( (1-p) \) of 70% that failure rate will be at the estimated worst possible value \( f_{iw} \). When the indifference point is reached, that is, when the designer is equally likely to take the “lottery” or stay with the “certain” alternative, a point in the single attribute utility function, \( U_i(f_i) = p \), is obtained.

The following equations shows the derivation of this result:

\[
U_i(f_i) = p \cdot U_i(f_{ib}) + (1-p) \cdot U(f_{iw})
\]

\[
U_i(f_{ib}) = p \cdot (1) + (1-p) \cdot (0)
\]

\[
U_i(f_{iw}) = p
\]

The value of \( p \) at which the designer will be indifferent is obtained by iterating through extreme values of \( p \).

The value of \( k_i \) is equal to the utility where the attribute \( f_i \) is at its best level, \( f_{ib} \) and all of the other attributes are at their worst levels; at this point \( U(f_{1w}, \cdots, f_{ib}, \cdots, f_{nw}) = k_i \).

The “certain” alternative shown in Fig. 5 represents a design alternative with attribute levels known with certainty, and the lottery represents a design with uncertain attribute levels. The lottery shows a probability \( p \) of 60% that the design has the estimated best attribute levels \( f_{1w} = 9.419 \times 10^{-3} \text{ fr Mh}^{-1} \); \( f_2 = 0.4 \text{ m} \) and a probability \( (1-p) \) and a probability of 40% that the design will exhibit the estimated worst attribute levels \( f_{1w} = 12.390 \times 10^{-3} \text{ fr Mh}^{-1} \); \( f_2 = 0.8 \text{ m} \).

The value of \( k_i \) is equal to the value of \( p \) corresponding to the indifference point; see the following equations for the derivation of this result:

\[
U(f_{1w}, \cdots, f_{ib}, \cdots, f_{nw}) = p \cdot U(f_{ib}) + (1-p) \cdot U(f_{iw})
\]

\[
U(f_{1w}, \cdots, f_{ib}, \cdots, f_{nw}) = p \cdot (1) + (1-p) \cdot (0)
\]

\[
U(f_{1w}, \cdots, f_{ib}, \cdots, f_{nw}) = k_i
\]

Details of the certainty equivalent method can be found in Keeney et al. [16].
CERTAIN

Design with certain failure rate \( f \)

\[
f = \frac{10.0 \times 10^{-6}}{\text{fr Mh}^{-1}}
\]

LOOTERY

Design with uncertain failure rate

\[
p = 0.3
\]

\[
p = 0.7
\]

\[
ap = 12.39 \times 10^{-6} \text{fr Mh}^{-1}
\]

Fig. 5. An example of the lottery questions used in the certainty equivalent method to assess the single attribute scaling constant \( k \) for failure rate \( (f_1) \).

| TABLE II |

THERMAL CHARACTERISTICS OF THE HEATED ELEMENTS CONSIDERED IN THE PRESENT CASE STUDY |

<table>
<thead>
<tr>
<th>Element</th>
<th>Height (cm)</th>
<th>( q_i ) (W m(^{-2}))</th>
<th>( B_i ) (fr Mh(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>400</td>
<td></td>
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<td>3</td>
<td>0.2</td>
<td>1600</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>1600</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>400</td>
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<tr>
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<td>0.4</td>
<td>400</td>
<td></td>
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<tr>
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<td>0.4</td>
<td>1600</td>
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</tr>
<tr>
<td>8</td>
<td>0.4</td>
<td>1600</td>
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</tr>
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</table>

and a minimum grid spacing of 0.05. This grid allows the prediction of the maximum temperature of the heated elements within 1% of those obtained using the most refined grid with 75% less CPU time. For the case study, the results of using the two multiobjective optimization strategies under consideration, that is, Pareto optimization and MUA, are presented.

Before presenting and discussing the results associated with these two multiobjective optimization strategies, a thermal placement problem with known optimal solutions is first addressed (Validation run).

A. Validation Run

Consider the placement of the heated elements listed in Table III so that the total failure rate of the system (2) is minimized. Observe that the heated elements generate the same heat flux and that the maximum temperature of the heated elements is only a function of their position along the channel. Under these conditions, it can be shown (Queipo et al. [14]) that the optimal arrangement requires placing the heated elements in descending order of thermal sensitivity. Hence, optimal sequences are, for example, 3 4 7 2 6 8 1 5, 7 4 3 8 6 2 5 1, 4 3 7 6 2 8 1 5, etc. The total number of possible arrangements is 40,320 (8!), and there are 72 (3 \( \times \) 3 \( \times \) 2) optimal solutions representing 0.18% of the total solution space.

| TABLE III |

THERMAL CHARACTERISTICS OF THE HEATED ELEMENTS CONSIDERED IN THE VALIDATION RUN |

<table>
<thead>
<tr>
<th>Element</th>
<th>Height (cm)</th>
<th>( q_i ) (W m(^{-2}))</th>
<th>( B_i ) (fr Mh(^{-1}))</th>
</tr>
</thead>
<tbody>
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<td>200</td>
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<td>6</td>
<td>0.2</td>
<td>400</td>
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<td>7</td>
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<tr>
<td>8</td>
<td>0.2</td>
<td>400</td>
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B. Control Parameters for the Genetic Algorithm

The population size was taken as seven (as in Queipo et al. [12]) and the number of generations was specified as nine. Numerical simulations of the genetic algorithm were conducted for a range of crossover rates, mutation rates and scaling factors. The crossover rate and mutation rates considered were between 0.1 and 0.9 with increments of 0.1 (with the restriction of mutation rates lower than crossover rates). The scaling factors were taken between 1.0 and 3.0 with increments of 1.0. The present genetic algorithm exhibited a robust behavior. At the end of nine generations, a significant number of combinations of crossover rate, mutation rate and scaling factor \((C, M, S)\) generated optimal solutions. For example \((0.9, 0.2, 1.0), (0.9, 0.2, 1.0), (0.8, 0.4, 1.0), (0.6, 0.3, 1.0), (0.4, 0.1, 1.5), (0.8, 0.4, 1.5), (0.4, 0.1, 2.0), \) and many others. All the results reported in this section and throughout the study correspond to a crossover rate of 0.9, a mutation rate of 0.4 and a sigma scaling factor equal to 1.0.
Fig. 6. Temperature isocontours corresponding to the validation run. The figure shows 15 equally spaced contours between 302 and 330 K.

Table IV shows the ten best arrangements uncovered by the genetic algorithm. Each entry in the table shows a given arrangement of components, its failure rate, the number of the generation in which it appeared and the corresponding number of objective function evaluations. The best elements correspond to the sequences 47368215 (ninth generation—fifty seven objective function evaluations) and 37468215 (eighth generation—fifty four objective function evaluations) with failure rate of 5.643e-03 fr Mh⁻¹. These sequences have the heated elements positioned in decreasing order of thermal sensitivity and are optimal solutions. Note that the expected number of objective function evaluations to randomly find an optimal solution is given by the number of possible arrangements (8!) divided by the number of optimal solutions (3!×3!×2!) and equal to 560. The genetic algorithm found an optimal solution using an order of magnitude fewer objective function evaluations.

V. RESULTS AND DISCUSSION

This section addresses the more complex situation where all the heated elements are different in their heat generation rates or their thermal sensitivities and the optimization criteria include both thermal and nonthermal optimization criteria. As previously discussed, in the case of multiobjective optimization there is not a general definition of the optimal values and no single best approach for solving these problems. As a result, different philosophies and methodologies, such
Fig. 8. Best arrangements obtained by the CSGA using Pareto optimization for the cases of \((w_3, w_4): (1.0, 0.0), (0.5, 0.5), \) and \((0.0, 1.0)\). Top: \((1.0, 0.0)\); 43725618, 332 K; 340 K; 345 K; 354 K; 341 K; 350 K; 331 K; 335 K; 330 K; 339 K; 337 K; 355 K; 349 K; 350 K. Middle: \((0.5, 0.5)\); 42371658, 332 K; 330 K; 339 K; 337 K; 340 K; 345 K; 354 K; 341 K; 350 K; 331 K; 335 K; 330 K; 339 K; 337 K; 355 K; 349 K; 350 K. Bottom: \((0.0, 1.0)\); 12345678, 332 K; 334 K; 327 K; 340 K; 345 K; 354 K; 356 K; 349 K; 347 K; 350 K. Maximum temperatures of the heated elements are also shown.

Fig. 9. Single attribute utility function for failure rate.

as Pareto optimization and MUA, co-exist for addressing optimization problems with multiple objectives.

The thermal and nonthermal optimization criteria correspond to the minimization of the failure rate of the system computed using the Arrhenius relation and of the total wiring length according to an interconnectivity matrix. The present interconnectivity requirement is that the heated elements identified with numbers between 1 and 4 inclusive and those identified with numbers between 5 and 8 must be wired among themselves, respectively. The total interconnectivity length and total failure rate of the arrangements of heated elements are denoted by the functions \(\lambda (2)\) and \(g (3)\), respectively.

Table II presents a description of the thermal characteristics of the heated elements under consideration. The control parameters of the genetic algorithm adopted were exactly those selected in the Validation run.

Fig. 10. Single attribute utility function for wiring length.
Fig. 11. Temperature isocontours associated with the best arrangement (42,371,568) uncovered by the CSGA. The figure shows 15 equally spaced contours between 302 and 352 K. (MUA).

Fig. 12. Maximum temperatures of the heated elements associated with the best arrangement uncovered by the CSGA; \( f_1 = 9.725 - 0.03 \text{ fr Mh}^{-1} \) and \( f_2 = 0.52 \text{ m} \). (MUA).

A. Pareto Optimization

Solutions expected to belong to the Pareto optimal set are calculated using the weighting method (Balachandran et al. [15]) which converts the multiobjective problem to a single objective problem, in which the function to be optimized is the weighted sum of the individual objective functions. In this case, the function \( f \) to be minimized has the form

\[
 f = w_\lambda \cdot \lambda_{\text{total}} + w_\delta \cdot (C_\delta g) \tag{17}
\]

where \( C_\delta \) represents a scaling factor, calculated for each generation in order to render the average contribution of the interconnectivity term in the sum comparable in magnitude to the average contribution due to the total failure rate. The coefficients \( w_\lambda \) and \( w_\delta \) are weighting factors representing the relative importance of the optimization criteria, with \( w_\lambda + w_\delta = 1 \). In this work, three points in the Pareto optimal set (including the extremes of the set and a situation where the optimization criteria are considered to be equally important) are sought (\( w_\lambda, w_\delta \): (1.0, 0.0), (0.5, 0.5), and (0.0, 1.0).

The failure rate and interconnectivity length associated with the three Pareto optimal solutions are plotted in Fig. 7. The solution corresponding to weighting factors (1.0, 0.0) has a failure rate of 9.419 fr Mh\(^{-1}\) and an interconnectivity length of 0.64 m. This solution was found after five generations (thirty one objective function evaluations) and corresponds to a situation where the minimization of the failure rate is the sole optimization criterion. A solution corresponding to the other extreme of the Pareto optimal set; that is, the situation where the minimization of the wiring length is the only optimization criterion (\( w_\lambda = 0 \) and \( w_\delta = 1.0 \)) was selected by inspection of the interconnectivity requirement. The optimal solution selected for this case was the sequence 12345678 with a failure rate of 10.770 fr Mh\(^{-1}\) and an optimal wiring length of 0.4 m. The Pareto optimal solution associated with the situation with equal weighting factors was obtained after nine generations (sixty six objective function evaluations) and corresponds to the sequence 42371658. The aforementioned sequence has a failure rate of 9.600 fr Mh\(^{-1}\) and an interconnectivity length of 0.52 m. Note that the Pareto optimal solutions under consideration are in fact nondominated solutions (see Fig. 8), with their failure rate and interconnectivity length varying by up to 14 and 60%, respectively.

B. Multiattribute Utility Analysis

This section discusses the solution of the case study using the single attribute utility functions for failure rate (\( f_1 \)) and wiring length (\( f_2 \)) shown in Figs. 9 and 10, respectively. Figs. 9 and 10 correspond to quadratic polynomials that interpolate the points \( [f_1, U(f_1)]: (9.4e - 03 \text{ fr Mh}^{-1}, \ 1.0) \).
(100e−03 Fr Mh⁻¹, 0.7), (124e−03 Fr Mh⁻¹, 0.0); and the points \( f_2, U(f_2) \): (0.4 m, 1.0), (0.6 m, 0.7), (0.8 m, 0.0), respectively.

The scaling factors reflecting acceptable tradeoffs between attributes, are given as \( k_1 = 0.6 \) (failure rate) and \( k_2 = 0.4 \) (wiring length). Both, the utility functions and the scaling factors are assumed to have been obtained with the participation of the designer and the certainty equivalent method discussed in a previous section. The function to be maximized is given by (10) with the aforementioned utility functions and scaling factors.

The ten best arrangements obtained when using the MUA approach are shown in Table V. The best arrangement corresponds to the sequence 42371568 with a failure rate and wiring length of 0.725e−03 Fr Mh⁻¹ and 0.52 m, respectively. The best arrangement was found after nine generations and 65 function evaluations.

The temperature isocontours as well the maximum temperature on each of the heated elements corresponding to the sequence 42371568 are shown in Figs. 11 and 12, respectively.

Note that this approach provides the "best" solution with a single coupled execution of the fluid and heat transfer solver and the genetic algorithm provided the utility functions \( U(f_2) \) and the scalar constants \( (k_2) \) are available. In addition, this approach could be used to identify the "best" solution among the Pareto optimal solutions found (those obtained in the previous section) by computing the utility value of each of the Pareto optimal solutions and selecting the solution with highest utility value.

VI. CONCLUSION

A model for the problem of optimal placement of electronic components on printed wiring boards subject to thermal and nonthermal optimization criteria has been formulated and solved using a methodology based on three components:

1) a fluid and heat transfer solver for the prediction of the maximum temperature of the heated elements;
2) a multiobjective optimization strategy for the scalarization of the vector of design objectives;
3) a genetic algorithm for the search of optimal or near optimal solutions.

The multiobjective optimization strategy embedded in the solution methodology is flexible enough to account for two extreme situations (no knowledge/knowledge) regarding the knowledge of the preference structure of the designer by using Pareto optimization and MUA.

The solution methodology shows promise as an effective and efficient tool for providing optimal or near-optimal solutions for electronic component placement problems where both thermal and nonthermal optimization criteria are of interest under rather general conditions regarding component geometries, heat generation rates and thermal sensitivities.

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Nestor V. Queipo received the B.Sc. degree in civil engineering and the M.Sc. degree in computer science from the University of Zulia, Venezuela, and the Ph.D. degree in mechanical engineering from the University of California at Berkeley.

He was a member of the faculty of the Aerospace and Mechanical Engineering Department, University of Arizona, Tucson, and is currently Associate Professor and Director of the Applied Computing Institute, University of Zulia. His research interests include prediction of transport phenomena, analysis of complex data, surrogate modeling and optimal design of complex systems, and nonlinear process control.

Joseph A. C. Humphrey received the Ph.D. and D.Sc. Eng. degrees from the Imperial College of Science, Technology, and Medicine, London University, London, U.K., in 1977 and 1997, respectively.

He became Dean and Professor of Mechanical Engineering at Bucknell University, Lewisburg, PA, in August 1997. From 1977 to 1995, he was on the faculty of the Department of Mechanical Engineering, University of California, Berkeley, and from 1995 to 1997, he was Head of Aerospace and Mechanical Engineering at the University of Arizona, Tucson. His teaching and research interests are in fluid mechanics and transport phenomena, with emphasis on the application of experimental and computational methodologies to problems ranging from multiphase flows through complex geometries to microsensing in arthropods.

Dr. Humphrey is an active member of several major engineering societies, editorial, and industrial boards, and the recipient of various awards and honorary professorships, including Fellow of the ASME and Fulbright Fellow.

Alfonso Ortega (M’96) received the B.S. degree from the University of Texas, El Paso, in 1976 and the M.S. and Ph.D. degrees from Stanford University, Stanford, CA, in 1978 and 1986, respectively, all in mechanical engineering.

He was a Member of the Technical Staff at Sandia National Laboratories, Albuquerque, NM, in the Fluid and Thermal Sciences Division, from 1978 to 1981, and in the Geothermal Research Division, from 1986 to 1988. In 1988, he joined the faculty of the Department of Aerospace and Mechanical Engineering, University of Arizona, Tucson, where he is currently Associate Professor. His research is in the area of experimental and numerical convective heat transfer in complex forced and buoyant flows, especially those arising in air cooling of electronics and cooling of turbomachinery. He established the AME Heat Transfer Laboratory at the University of Arizona in 1988 and is a member of the University of Arizona Center for Electronics Packaging Research. He is the author of numerous technical papers and book chapters on air cooling of electronics, and is a frequent lecturer on the subject, both in the United States and abroad. He is currently Associate Technical Editor of the ASME Journal of Electronic Packaging. He currently holds positions as Guest Researcher in the Semiconductor Electronics Division at the National Institute of Standards and Technology, Gaithersburg, MD and Visiting Guest Scientist at the Burger Center for Fluid Mechanics at the Technical University of Delft, Delft, The Netherlands.

Dr. Ortega has served as Chairman of the ASME Heat Transfer Division K-16 Committee on Heat Transfer in Electronic Equipment from 1993 to 1997, General Chairman of the 1994 IEEE SEMITHERM Conference, and General Chairman of the 1994 ASME-IEEE IThERM Conference. In 1990, he was named a National Science Foundation Presidential Young Investigator.