A GENERALIZED SURROGATE CONTACT MODEL FOR DYNAMIC SIMULATIONS WITH ANATOMIC JOINTS

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INTRODUCTION

Musculoskeletal computer models are useful for estimating physiological quantities that cannot be measured experimentally [1], and designing new medical devices and rehabilitation approaches [2]. Lack of articular contact in musculoskeletal computer models can lead to inaccurate prediction of quantities influenced by the interactions between muscles, ligaments, and bones. However, unlike engineering joint models, articular contact models require repeated evaluation of surface geometry. In dynamic contact simulations, these geometry evaluations consume the vast majority of the CPU time.

Recently, the first dynamic knee simulation utilizing a surrogate contact modeling approach has been reported. This approach reduced the simulation time of one gait cycle to less than one and half minutes [3]. Unfortunately, because the surrogate model was implemented using built-in software functions, it was regenerated each time a value of contact force was needed by the numerical integrator. In addition, the technique used in that work was only suitable for joints constrained to planar motion.

This study proposes a generalized surrogate modeling technique to speed up dynamic musculoskeletal simulations incorporating articular surface contact. This approach involves using knowledge of contact mechanics to fit a computationally-cheap surrogate model to data points generated by a computationally-costly contact model. The goal was to determine whether a dynamic simulation performed with the generalized surrogate contact model could closely reproduce the planar motions and contact forces produced by an elastic foundation (EF) contact model (i.e., the computationally-costly contact model) but with substantially reduced computational cost. The proposed surrogate contact model can be viewed as a generalized Hertzian contact model where the stiffness and exponent in the model are allowed to vary.

METHODS

A 3D artificial knee contact model created from an Osteonics 7000 cruciate-retaining knee implant (Stryker Howmedica Osteonics, Inc, Allendale, NJ) was used to compare contact predictions using EF and surrogate contact models. Dynamic equations were derived for the femoral component moving in the sagittal plane with respect to a fixed tibial insert. Sagittal plane rotation (i.e., $z_r$) was prescribed while superior-inferior (SI) translation (i.e., $y_t$) and anterior-posterior (AP) translation (i.e., $x_t$) were predicted by numerically integrating the equations of motion using the stiff solver ode15s available in Matlab (The Mathworks, Natick, MA). The prescribed flexion angle and applied loads in the SI and AP directions were taken from a planar version of a dynamic simulation performed in a previous study using the same implant model [4].

Equal and opposite contact loads were applied to the femoral component and the tibial insert in the dynamic simulation using two different articular contact models. The first one was an EF contact model employing a fine element grid of 100 x 100 on the medial and lateral sides of the tibial insert. The second one was a surrogate contact model created from sample points generated by the EF contact model. Contact forces, joint motion, and CPU time determined using an EF contact model were compared to those found using a surrogate contact model. For both contact models, the dynamic simulation was performed on a 2.8 GHz Pentium IV PC.

To develop the surrogate contact model, we needed to determine a sampling scheme within the design space ($x_t$, $y_t$, and $z_r$) to provide data for fitting the surrogate model. We choose the Latin hypercube sampling (LHS) sampling scheme, which places at most one sample point in each row and column of the multi-dimensional design space. To maximize the quality of the resulting surrogate model fit while minimizing the number of sample points, we needed to decide which
In this equation, \( k \) is the contact stiffness, \( d \) is the interpenetration, \( yti \) is the initial contact (i.e., \( yt \) with 10N load) and \( n \) is the contact exponent. At each location on the LHS points, a pair of \( xt \) and \( zr \) were obtained using power fit in Matlab while \( d \) was the difference between current and initial \( yt \). Since the Hertian contact model is suitable only for normal contact force, Eq. (2) is needed for predicting \( Fx \).

\[
Fx = \text{ratio}(xt,zr)Fy \tag{2}
\]

where \( \text{ratio} \) is the average of 7 quotients obtained from 7 known values of \( Fx \) divided by \( Fy \).

After 1050 static analyses, Kriging [6] model and polynomial with Fourier harmonics model were used to fit \( k, n, yti, \) and \( \text{ratio} \) as functions of \( xt \) and \( zr \). Originally developed for geostatistics and spatial statistics, a Kriging model is the combination of a polynomial model with a correlation function. After evaluating several different combinations, a quintic polynomial plus the Gaussian correlation function were selected to construct Kriging model while poly-Fourier model was modeled by a cubic polynomial with one Fourier harmonic.

Each time the dynamic simulation required a value for \( Fx \) and \( Fy \), the surrogate contact models used the current values of \( xt \), \( yt \), and \( zr \) to perform the following calculations. First, new values of \( k, n, \) and \( \text{ratio} \) were calculated from their corresponding surrogate models using \( xt \) and \( zr \) as inputs. Next, \( d \) was calculated using \( xt \) and \( zr \), and the current value of \( yt \) as inputs. Finally, \( Fy \) was calculated from Eq. (1), which was used to calculate \( Fx \) using Eq. (2).

RESULTS

The dynamic simulation that used the generalized surrogate contact models closely reproduced the motions and loads predicted with the EF contact model (Fig.1). However, motion and force were matched more closely in the SI direction than in the AP direction. Furthermore, \( Fx \) predicted from Kriging model was slightly more accurate than in poly-Fourier model. While the dynamic simulation using the EF contact model required 74 minutes of CPU time, the simulation using surrogate contact models required less than 20 seconds.

DISCUSSION

Using a surrogate model to replace the original articular contact model possesses several advantages. First, generation of sample points for surrogate model construction requires repeated static rather than dynamic analyses. Moreover, instead of distributing the sampling points uniformly in the design space LHS sampling scheme was chosen for its practical application in high-dimensional design space problem. Second, the generalized surrogate modeling approach is applicable to most human joints, therefore multiple surrogate contact models can be embedded within any larger dynamic musculoskeletal model without incurring large increases in computational cost. Third, the use of design optimization which usually requires hundreds or even thousand of repeated dynamic simulations can be efficiently realized using surrogate modeling approach. Furthermore, the system to be optimized can be changed without the need to generate new surrogate models, facilitating optimizations involving full-body musculoskeletal models.

The primary limitations of this approach are that the calculation of \( Fx \) is dependent on the value of \( Fy \) computed from Eq. (1). Also, new surrogate contact models must be generated for any change in surface geometry or material properties. Also, friction effects are not included in this initial formulation.

**Figure 1:** Comparison of motions and loads predicted by the EF and two surrogate contact models. (a) AP (blue) and SI (red) translation. (b) \( Fx \) (blue) and \( Fy \) (red) forces.

CONCLUSIONS

This paper has presented a generalized surrogate modeling technique for efficient dynamic simulation of anatomic joints. The technique is demonstrated using a dynamic simulation of a sample knee replacement constrained to planar motion. Two different surrogate contact models produced comparable accuracy with orders of magnitude improvement in computational speed. Independent calculation of \( Fx \) and the extension of this approach to high-dimensional problems are topics for future research efforts.

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