Model-Based Adaptive-Predictive Control and Optimization of SAGD under Uncertainty

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Abstract

When it comes to SAGD optimization, two of the biggest challenges are controlling subcool to achieve conformance (a uniform growth of the steam chamber along the complete length of the well pair), and maximizing an economic performance measure, such as net present value (NPV); both desirable outcomes are not necessarily associated with the same values of the operational parameters (e.g., injection rates). Overcoming these challenges is necessary for achieving optimum SAGD performance, but this may be difficult through common operating policies, e.g. injecting steam at an empirically specified constant rate, considering well-known features of SAGD processes, e.g., complex dynamics (nonlinear, slow, high order, time varying, potentially highly heterogeneous reservoirs), operational constraints, model uncertainty and measurement noise. In the context of this work, the aforementioned challenges are both formulated as optimization problems of adjusting injection rates in order to optimize a particular objective function (e.g. minimizing subcool error or maximizing NPV). To address these challenges, this paper presents a nonlinear model-based adaptive-predictive control approach, alternative to the classical Proportional-Integral-Derivative (PID), for subcool control and NPV optimization of SAGD processes under uncertainty. Using case studies with an idealized heterogeneity pattern (subcool control) and multiple geological realizations based on logs from the Orinoco Belt region (NPV optimization), the proposed approach was compared with a decentralized PID for subcool control, in terms of response speed (mean square error and reaching time), steady state behavior (settling time, measured subcool mean and standard deviation) and control energy spending. While the proposed approach offered a slower response (not a critical issue in terms of oil recovery), it significantly outperformed the PID control during steady state and in control energy spending. On the other hand, the effectiveness for NPV optimization under uncertainty was demonstrated against constant steam injection strategies considering mean NPV, steam injection, water produced, and SOR, and when modeling alternative risk aversion scenarios.

Keywords Thermal recovery · SAGD optimization · Subcool control · PID control · MPC

Introduction

By 2035 the energy demand is expected to grow approximately 35%, with around 54% of such demand being fulfilled by oil and gas (British Petroleum 2014). Considering that about 70% of the world oil
reserves correspond to heavy oil, extra-heavy oil and bitumen (Alboudwarej et al. 2006), thermal recovery methods should play a key role in satisfying such demand. Among these recovery methods, steam-assisted gravity drainage (SAGD) has shown the potential to achieve recovery factors in the range of 50% to 75% (Egermann et al. 2001). Fig. 1 illustrates a typical SAGD configuration in which an upper steam injection well and a lower production well are drilled a few meters apart (vertically); as the injection is carried on, a steam chamber is formed around the injector with oil drained through the lower producer as a result of viscosity reduction and gravity effects (Butler 1991).

During the operation of SAGD processes, two of the biggest challenges are: (i) achieving conformance, i.e., a uniform growth of the steam chamber along the complete length of the well pair to keep it well drained, and ii) maximizing an economic performance measure, such as, net present value (NPV). The former requires strategies for subcool control so that the liquid does not accumulate over the producing well but neither is steam produced (Edmunds 1998), while the latter accounts for economic and efficiency issues such as operating costs, oil prices and steam-oil ratio (SOR). Subcool makes reference to the difference between the saturation temperature and the actual temperature of water at the producer; on the other hand, SOR represents the volume of steam necessary to produce one volume unity of oil and is a measure of recovery efficiency (Egermann et al. 2001).

Overcoming these challenges is necessary for achieving optimum SAGD performance, but this may be difficult through common operating policies, e.g. injecting steam at a empirically specified constant rate, considering well-known features of SAGD processes, such as: (i) multiple injection (input variables) and production (output variables) segments, (ii) complex dynamics, i.e., nonlinear, slow, high order, time varying, potentially highly heterogeneous reservoirs, (iii) operational constraints, and (iv) model uncertainty and measurement noise. Subcool control for tackling conformance has been essentially addressed through the conventional Proportional-Integral-Derivative (PID) approach (Hin-Sum Law et al. 2013, Stone et al. 2013, Stone and Bailey 2014). While PID will regularly drive and keep the subcool relatively close to the set-point, the control performance might not be completely satisfactory considering the SAGD process above-referenced complex features. On the other hand, previous works regarding SAGD optimization, in general, exhibit one or more of the following limitations:


b. Potentially inaccurate proxy model. The proxy or surrogate model used in a surrogate-based
optimization procedure may not be accurate enough to describe the complex dynamics of a SAGD process (Fedutenko et al. 2013).


This paper presents a computationally efficient nonlinear model-based adaptive-predictive control strategy as an alternative to PID, for subcool control and NPV optimization of SAGD processes that accounts for complex reservoir dynamics, geological uncertainty, measurement noise and operational constraints.

**Problem formulation**

For a SAGD reference configuration, with injector and producer wells with \( N \) completed segments each, and a production horizon comprising \( K \) sampling instants, there are two problems of interest associated with the aforementioned challenges: subcool control and maximization of net present value (NPV). In the context of this work, both problems are formulated as optimization problems, i.e. in both cases it is desired to find the steam rate to be injected at each of the completed segments of an injector well at each sampling instant, to optimize a particular performance measure (minimize error during subcool control or maximize net present value at the end of a production horizon), while accounting for complex reservoir dynamics, geological uncertainty, measurement noise and operational constraints. Furthermore, both pose control problems because the proposed approach for NPV optimization involves finding steam injection rates to reach the production set-points that maximize NPV. Both problems are now formally formulated:

**Subcool control**

The subcool control problem can be formally stated as:

Find \( u \) which

\[
\min f_{\Delta T}(\epsilon, u)
\]

subject to:

\[
0 \leq u_n(k) \leq 1
\]

for \( n = 1, ..., N \) and \( k = 1, ..., K \)

while honoring the reservoir dynamics described by the models:

\[
\Delta \hat{T}_n(k) = g_n(\Delta T_n(k - 1), ..., \Delta T_n(k - p), u_n(k - 1), ..., u_n(k - p))
\]

where:

- \( f_{\Delta T} \): represents a function of the subcool error and the control energy
- \( \epsilon \): vector of errors defined by \([\epsilon_1(0) \ldots \epsilon_N(0), \ldots, \epsilon_1(K - 1) \ldots \epsilon_N(K - 1)]\), with \( \epsilon_n(k) = \Delta T_n^0(k) - \Delta T_n(k) \) where \( \Delta T_n^0(k) \) is the subcool set-point (equal for all completed segments) at the \( k \)th sampling instant, and \( \Delta T_n(k) \) represents the measured subcool in the \( n \)th completed segment of the producer well at the \( k \)th sampling instant
- \( u \): control sequence defined by \([u_1(0) \ldots u_N(0), \ldots, u_1(K - 1) \ldots u_N(K - 1)]\) function of \( \Delta \hat{T}_n(k) \);
  - with \( u_n(k) \) the opening of the valve in the \( n \)th completed segment at the \( k \)th sampling instant
- \( \Delta \hat{T}_n(k) \): estimate of \( \Delta T_n(k) \)
- \( g_n \): nonlinear function
- \( p \): positive integer

**NPV optimization under uncertainty**

On the other hand, given \( M \) geostatistical realizations of the petrophysical properties, the NPV maximization problem can be formally expressed as

Find \( u \) which
\begin{align*}
\max_{\mathbf{q}, \mathbf{u}, \theta} f_{NPV}(\mathbf{q}, \mathbf{u}, \theta) \\
\text{subject to:}
\end{align*}

0 \leq u_n(k) \leq 1 
\text{for } n = 1, \ldots, N \text{ and } k = 1, \ldots, K

while honoring the reservoir dynamics described by the models:

\begin{align*}
\hat{q}_m^o(k) &= g_m^o(q_m^o(k-1), \ldots, q_m^o(k-z), \mathbf{u}(k-1), \ldots, \mathbf{u}(k-z), \theta_m) \\
\hat{q}_m^g(k) &= g_m^g(q_m^g(k-1), \ldots, q_m^g(k-z), \mathbf{u}(k-1), \ldots, \mathbf{u}(k-z), \theta_m) \\
\hat{q}_m^w(k) &= g_m^w(q_m^w(k-1), \ldots, q_m^w(k-z), \mathbf{u}(k-1), \ldots, \mathbf{u}(k-z), \theta_m)
\end{align*}

for \( m = 1, \ldots, M \)

where:

- \( f_{NPV} \): represents a function of the net present value
- \( \mathbf{q} \): vector defined by 
  \[ [q_1^o(1), q_1^o(2), \ldots, q_1^o(K), q_2^o(1), q_2^o(2), \ldots, q_2^o(K), \ldots, q_M^o(1), q_M^o(2), \ldots, q_M^o(K)] \]
  for \( m = 1, \ldots, M \) geostatistical realizations, with \( q_m^o(k) \), \( q_m^g(k) \) and \( q_m^w(k) \) the measurements of oil, gas and water production rates, respectively, at the \( k \)th sampling instant
- \( \mathbf{u} \): control sequence defined by 
  \[ [u_1(0) \ldots u_1(K-1) \ldots u_N(0) \ldots u_N(K-1)] \]
  function of \( \hat{q}_m^o(k), \hat{q}_m^g(k) \) and \( \hat{q}_m^w(k) \); with \( u_n(k) \) the opening of the valve in the \( n \)th completed segment at the \( k \)th sampling instant
- \( \theta \): vector defined by \( [\theta_1, \theta_2, \ldots, \theta_M] \), where \( \theta_m \) denotes the \( m \)th geostatistical realization
- \( \hat{q}_m^o(k), \hat{q}_m^g(k), \hat{q}_m^w(k) \): estimates of \( q_m^o \), \( q_m^g \) and \( q_m^w \)
- \( z \): a positive integer
- \( q_m^o, q_m^g, q_m^w \): nonlinear functions

**Proportional-Integral-Derivative control (PID) versus Model Predictive Control (MPC) in SAGD processes**

The basic structure of a continuous-time PID controller is given by (Åström & Hägglund 1995)

\begin{equation}
\mathbf{u}(t) = K\left( e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{d(e(t))}{dt} \right)
\end{equation}

where \( u(t) \) is the control input, \( e(t) \) is the error (difference between the set-point and the measured value of the output), \( K \) is the proportional gain, \( T_i \) is the integral time and \( T_d \) is the derivative time. In addition, several methods are available for discretizing the continuous-time version (9) for digital implementations. Due to its simple and intuitive structure, the classic PID and related methods are popular controllers, and a typical choice for general control applications. However, although PID will regularly “do the job”, i.e., will drive and keep the controlled variable (output) relatively close to the set-point, its performance in the context of SAGD might not be completely satisfactory considering the process above-referenced complex features.

On the other hand, model-based control refers to a set of strategies which explicitly use a process model for controller design (Ogunnaike and Ray 1994). There are two approaches for designing model-based controllers: (i) direct synthesis, such as, direct synthesis control, internal model control and generic model control, where the desired output behavior is specified as a trajectory and the controller is synthesized such that the output of the model exactly follows this trajectory and, (ii) optimization, e.g., model predictive control schemes-MPC, where the desired behavior is specified as an objective function, and the process
is used to determine the controller to minimize this objective. The MPC approach is a notable model-based control scheme considering its remarkable success in many complex industrial applications (Lee 2011). Any MPC approach consists of four basic elements:

1. Reference trajectory specification: this element specifies the desired target trajectory for the output of the model, e.g., a step to the new set-point.
2. Process output prediction: a discrete-time model (e.g. finite convolution, discrete state-space and discrete transfer functions) is used to predict the process output over an extended (future) time horizon with a pre-specified length, starting at the current time step. If the model is nonlinear, the scheme is known a Nonlinear Model Predictive Control (NMPC).
3. Control sequence prediction: the same model is used to calculate a sequence of control inputs that will minimize an objective function, which in general involves the error and the control expenditure in driving the output to the target, subject to specified operating constraints, along the future time horizon.
4. Error prediction update: since no model constitutes a perfect representation of the plant, the model prediction error (difference between the measured output of the process and the output of the model) is used to update future predictions.

Solution methodology

This section describes the proposed methodologies for solving the problems of interest, namely: subcool control and NPV optimization under uncertainty.

Subcool control

The proposed approach is based on predictive control and data assimilation and assumes subcool measurements are available at every completed segment of the producing well. This approach includes the execution of the following steps at each sampling instant: (1) identification and continuous update (data assimilation) of $N$ neural network-based ARMAX (NN-ARMAX) models which describe the dynamics of subcool at each of the completed segments of the producing well, (2) optimal linearization of the NN-ARMAX models at the particular operating point, (3) construction of a linear MIMO Kalman innovation model to account for measurement noise and model uncertainty, and (4) design of a predictive control strategy to adjust the valve opening in every segment of the injection well to reach the (same) subcool set-points at every segment of the producing well. A sampling period of one day is used for subcool control. Further detail of these steps is shown below:

Identification and continuous update of $N$ neural network-based ARMAX (NN-ARMAX) models, each describing the dynamics of subcool at a particular completed segment of the producing well

For the $n$th segment, the corresponding NN-ARMAX model forecasts the subcool $\Delta T_n(k)$ at such segment at the $k$th sampling instant, using $p$ previous values of this subcool, $p$ previous values of the opening of the valve in the corresponding injecting segment (input) and $p$ previous values of the innovation (estimation) error of the neural network. Hence, each neural network has an input layer with $I = 3 \times p$ units and one output unit with the identity as activation function. In addition, each network has a hidden layer with $J$ units with the hyperbolic tangent as activation function.

The training algorithm is the Online Sequential Extreme Learning Machine (OS-ELM) developed by (Lian et al. 2006), and the training data is normalized linearly to expand the interval $[-1,1]$, using the formula

$$\theta = -1 + 2\frac{\Theta - \theta_{\text{min}}}{\Theta_{\text{max}} - \theta_{\text{min}}}$$

where $\Theta$ and $\theta$ represent the original and normalized values, respectively, and $\Theta_{\text{max}}$ and $\Theta_{\text{min}}$ are the maximum
and minimum values, respectively, of the corresponding input or output. From (10), the normalization function:

$$\theta = \text{nor}(\theta) = \frac{2}{\theta_{\text{max}} - \theta_{\text{min}}} \theta - \frac{\theta_{\text{max}} + \theta_{\text{min}}}{2}$$

(11)

and the denormalization function

$$\theta = d\text{nor}(\theta) = \frac{\theta_{\text{max}} - \theta_{\text{min}}}{2} \theta + \frac{\theta_{\text{max}} + \theta_{\text{min}}}{2}$$

(12)

are obtained.

For the $k$th sampling instant, $\Delta T_n(k)$ is the subcool, $\Delta \hat{T}_n(k)$ is the estimate of $\Delta T_n(k)$, $u_n(k)$ ($n = 1, \ldots, N$) is the valve opening at the $n$th completed segment of the injection well, and $e_n(k) = \Delta T_n(k) - \Delta \hat{T}_n(k)$ is the innovation (estimation) error. Given the input vector

$$\Psi = [\Delta T_n(k - 1), \ldots, \Delta T_n(k - p), u_n(k - 1), \ldots, u_n(k - p), e_n(k - 1), \ldots, e_n(k - p)]^T \in \mathbb{R}^{p \times 1}$$

(13)

to the neural network, the response of the $j$th hidden unit is calculated as

$$h_j = \tanh(\nu_j^T \Psi), j = 1, \ldots, J$$

(14)

where

$$\Psi = [\text{nor}_{\Delta T_n}(\Delta T_n(k - 1)), \ldots, \text{nor}_{\Delta T_n}(\Delta T_n(k - p)), \text{nor}_{u_n}(u_n(k - 1)), \ldots, \text{nor}_{e_n}(e_n(k - 1)), \ldots, \text{nor}_{e_n}(e_n(k - p))]^T \in \mathbb{R}^{p \times 1}$$

(15)

and $\nu_j = [\nu_{j0}, \nu_{j1}, \ldots, \nu_{jl}]^T$ is the weight vector corresponding to the $j$th hidden unit, where $\nu_{j0}$ is the bias of such unit, and $\nu_{ji}$ ($i = 1, \ldots, I$) is the weight connecting the $i$th input unit and the $j$th hidden unit. In addition, the output of the network is given by

$$\Delta \hat{T}_n(k) = d\text{nor}_n(h^T w_n)$$

(16)

where $h = [1, h_1, \ldots, h_J]^T$ and $w_n = [w_{n0}, w_{n1}, \ldots, w_{nj}]^T$ is the vector containing the weights that connect the output unit to the hidden units, where $w_{n0}$ is the bias of the output unit, and $w_{nj}$ ($j = 1, \ldots, J$) is the weight that connects the $j$th hidden unit and the output unit.

**Optimal linearization of each NN-ARMAX at the operating point corresponding to the current sampling instant, to obtain a locally equivalent linear ARMAX model**

The NN-ARMAX model defined by (14) and (16) can be expressed in the simplified form

$$\Delta \hat{T}_n(k) = g_{\Delta T}(\Psi)$$

(17)

$g_{\Delta T}: \mathbb{R}^p \rightarrow \mathbb{R}$ is a nonlinear vector function. Given a particular operating point $\Psi = \bar{\Psi}$, it is desired to find a linear model which is locally equivalent to (17) around $\bar{\Psi}$. This model has the form

$$\Delta \hat{T}_n(k) = G^T \Psi$$

(18)

where $G = [\varphi_1, \varphi_2, \ldots, \varphi_I]^T$. Taylor’s linearization method has been the most commonly used for local linearization. However, this method only applies in the vicinity of an equilibrium point equal to the origin. Based on the optimal linearization approach proposed by Teixeira and Żak (1999), which overcomes the aforementioned shortcomings of Taylor’s method, $G^T$ is given by Canelón et al. (2009)

$$G^T = \nabla_{\Psi^T} g_{\Delta T}(\bar{\Psi}) + \frac{g_{\Delta T}(\bar{\Psi}) - \nabla_{\Psi^T} g_{\Delta T}(\bar{\Psi})\bar{\Psi}}{\bar{\Psi}^T \bar{\Psi}} \bar{\Psi}^T$$

(19)

where
\begin{equation}
\n\nabla_{\Psi} g_{\Delta T}(\Psi) = \left[ \frac{\partial g_{\Delta T}}{\partial \Psi_1}(\Psi) \quad \frac{\partial g_{\Delta T}}{\partial \Psi_2}(\Psi) \quad \ldots \quad \frac{\partial g_{\Delta T}}{\partial \Psi_I}(\Psi) \right] \tag{20}
\end{equation}

where \( \Psi_i \quad (i = 1,\ldots,I) \) is the \( i \)th component of \( \Psi \) and, according to the chain rule

\begin{equation}
\n\frac{\partial g_{\Delta T}}{\partial \Psi_1} |_{\Delta=\hat{\Delta}} = \frac{2}{\Theta_{\text{max}} - \Theta_{\text{min}}} \sum_{j=1}^{I} \left[ w_{nj} \text{sech}^2(\alpha_j) \frac{2v_{ji}}{\Theta_{\text{max}} - \Theta_{\text{min}}} \right] \tag{21}
\end{equation}

where \( \alpha_j = \sum_{i=0}^{J} v_{ji} \cdot n_{ji}(\hat{\Delta}_i) \). In (21), \( \Theta_{\text{max}}, \), \( \Theta_{\text{min}}, \) represent the maximum and minimum values, respectively, of the \( i \)th input, and \( \Theta_{\text{max}}, \), \( \Theta_{\text{min}}, \) are the maximum and minimum values, respectively, for the output. The optimal linear model has the same dynamics of the nonlinear model at the operating point, and minimum modeling error in the vicinity of such operating point (Canelón et al. 2009).

The linear model (18) can be equivalently expressed as the linear ARMAX model

\begin{equation}
\Delta \hat{T}_n(k) = -a_1^{(k)} \Delta T_n(k-1) - \cdots - a_p^{(k)} \Delta T_n(k-p) + b_1^{(k)} u_n(k-1) + \cdots + b_p^{(k)} u_n(k-p) + d_1^{(k)} e_n(k-1) + \cdots + d_p^{(k)} e_n(k-p) \tag{22}
\end{equation}

where the superindex \( k \) refers to the sampling instant.

**Construction of a multi-input multi-output (MIMO) Kalman innovation model, from the linear ARMAX models** From (22), the multi-input multi-output (MIMO) Kalman innovation model

\begin{equation}
\begin{aligned}
\dot{x}_o(k+1) &= A_o^{(k)} x_o(k) + B_o^{(k)} u(k) + K_o^{(k)} e_n(k) \\
\Delta \hat{T}_n(k) &= C_o^{(k)} x_o(k)
\end{aligned} \tag{23}
\end{equation}

is constructed, where \( x_o(k) \in \mathbb{R}^{p \times 1} \) is the vector of estimated states, \( u(k) \in \mathbb{R}^I \) is the input vector, \( e(k) \in \mathbb{R}^p \) is the vector of innovation errors and \( \hat{y}(k) \in \mathbb{R}^p \) is the estimated output at sampling time \( k \),

\begin{equation}
A_o^{(k)} = \begin{bmatrix}
-a_1^{(k)} & 1 & 0 & \cdots & 0 \\
-a_2^{(k)} & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-a_p^{(k)} & 0 & 0 & \cdots & 0
\end{bmatrix} \in \mathbb{R}^{p \times p}, \quad B_o^{(k)} = \begin{bmatrix}
b_1^{(k)} \\
b_2^{(k)} \\
\vdots \\
b_p^{(k)}
\end{bmatrix} \in \mathbb{R}^{p \times 1}, \quad K_o^{(k)} = \begin{bmatrix}
d_1^{(k)} - a_1^{(k)} \\
d_2^{(k)} - a_2^{(k)} \\
\vdots \\
d_p^{(k)} - a_p^{(k)}
\end{bmatrix} \in \mathbb{R}^{p \times 1}
\end{equation}

\( C_o = [1 \ 0 \ \cdots \ 0] \in \mathbb{R}^{1 \times p} \). It is important to mention that it is not required to know the true order of the system to build the Kalman model (23–24).

**Design of a predictive control strategy to adjust the valve opening in every segment of the injection well to reach the (same) subcool set-points at every segment of the producing well** The MPC strategy proposed by (Wang 2009) is used to determine the valve openings corresponding to the current sampling instant. Such MPC strategy determines \( N_c \) future control inputs in order to minimize

\begin{equation}
J = (R_s - Y)^T \bar{Q} (R_s - Y) + \Delta U^T \bar{R} \Delta U \tag{25}
\end{equation}

along a prediction horizon of \( N_p \) future samples. In (25) the vector

\begin{equation}
Y = \begin{bmatrix} y(k+1|k), y(k+2|k), \ldots, (k+N_p|k) \end{bmatrix}^T \tag{26}
\end{equation}

contains \( N_p \) future values of the output, the vector

\begin{equation}
\Delta U = [\Delta u(k), \Delta u(k+1), \ldots, \Delta u(k+N_c-1)]^T \tag{27}
\end{equation}

contains \( N_c \) future variations of the control input. \( R_s^T = [1 \ 1 \ \cdots \ 1] r(k) = \bar{R} \in \mathbb{R}^{N_c \times N_c} \) is a block diagonal matrix which penalizes the variations of the control input and \( \bar{Q} \in \mathbb{R}^{N_p \times N_p} \) is a block diagonal matrix which penalizes the error in the subcool. It is expected that predictive control shows a good performance in systems with slow dynamics. It can be shown that (Wang 2009)
\[
\Delta U = (\phi^T \widehat{Q} \phi + \widehat{R})^{-1} \phi^T \widehat{Q} [R_s - F \widehat{x}_o(k)]
\]

where \(\widehat{x}_o(k)\) is the state estimated by the Kalman model (23–24), 
\[
\phi = \begin{bmatrix}
    C B & 0 & \cdots & 0 \\
    C A B & C B & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    C A^{n_p-1} B & C A^{n_p-2} B & \cdots & C A^{n_p-K} B
\end{bmatrix}, \quad F = \begin{bmatrix}
    C \\
    C A \\
    \vdots \\
    C A^{n_p}
\end{bmatrix}, \quad A = \begin{bmatrix}
    A_o & 0^T \\
    C_o A_o & 1
\end{bmatrix}, \quad B = \begin{bmatrix}
    B_o \\
    C_o B_o
\end{bmatrix}
\]

At every sampling instant only \(\Delta u(k)\) is chosen from \(\Delta U\), and the control input corresponding to the current sampling instant is given by
\[
u(k) = u(k-1) + \Delta u(k)
\]

Note that \(N\) MPC strategies are simultaneously designed to determine the valve openings (control inputs) that should be implemented at every completed segment of the injection well. Then, the procedure is repeated for the next sampling instant.

**NPV optimization under uncertainty**

The methodology proposed for NPV optimization comprises two stages: (1) optimization, to determine the well oil production rates (set-points) that maximize a function of the NPV, and (2) control, to determine the valve opening at the segments of the injection well to reach oil production set-points. This methodology is implemented using a dual sampling period: one year (long sampling period) for executing the optimization stage and one day (short sampling rate) for the control stage. In other words, at the beginning of every year the optimization stage is executed to find one set-point of oil production for each year left in the production horizon, and then the control stage is run to find the daily opening of the valves in the segments of the injecting well for one year that will make the oil production rate reach the corresponding set-point. Now, these stages are described:

**Optimization stage** Given \(M\) equiprobable geological realizations, the optimization problem can be stated as: find \(Y\) desired oil production rates, where \(Y\) is the number of years left in the producing horizon, which maximize the objective function (Gossuin et al. 2011)

\[
\max f_{NPV} = \mu - \lambda \sigma
\]

subject to constraints (5) and the reservoir dynamics (6, 7 and 8); where \(\mu\) is the mean of the cumulative NPV among all realizations, determined as,

\[
\mu = \frac{\sum_{m=1}^{M} NPV_m}{M}
\]

\(\sigma\) is the standard deviation of the NPV, determined as,

\[
\sigma = \sqrt{\frac{\sum_{m=1}^{M} (NPV_m - \mu)^2}{M}}
\]

and \(\lambda\) represents a risk aversion factor that, assuming the NPV samples are normally (Gaussian) distributed, can be associated to confidence levels typically used in standard probability distributions; so, for example, if \(\lambda = 1\) it can be interpreted that there is an 84.1\% chance that the actual value of NPV will be higher than that provided by the optimization of the objective function (30). Note that the set-points of oil production rates will be common for the \(M\) realizations, thus guaranteeing that the resulting optimum values can be implemented.

In (31) and (32), \(NPV_m\) is the NPV corresponding to the \(m\)th realization, given by,

\[
NPV_m = \sum_{k=1}^{K} \left[ \frac{(q_o^d (k) p_o + q_g^d (k) p_g - q_m^w (k) c_w - q_m^r (k) c_g) \Delta t_k}{(1+\beta)^{\frac{k \Delta t_k}{365}}} \right]
\]

where \(q_o^d (k)\), \(q_g^d (k)\) and \(q_m^w (k)\) are the daily production of oil [STB/D], gas [MSCF/D] and water [STB/D], respectively, and \(q_m^r (k)\) is the steam injection rate [STB/D], all for the \(m\)th realization at the \(k\)th
sampling time; \( P_o \) [USD/STB] and \( P_g \) [USD/MSCF] are the net income due to oil and gas production, respectively, \( C_w \) [USD/STB] is the cost of handling the water production, \( C_s \) [USD/STB] is the steam injection cost; \( r(k) \) is the tax rate at the \( k \)th sampling instant; \( \Delta t_k \) is the time step size in days; \( \beta \) is the annual discount factor; and \( K \) is the production horizon in days.

The algorithm used in numerically solving the aforementioned constrained optimization problem was the well-known interior point method (Byrd et al. 1999, Byrd et al. 2000, Waltz et al. 2006). During the optimization process \( \tilde{q}_m^o(k), \tilde{q}_m^g(k) \) and \( \tilde{q}_m^w(k) \) are estimated using three Kalman innovation models (23–24) one model for each production rate. Each Kalman model is constructed according to steps 1 to 3 of the methodology for subcool control, from a NN-ARMAX model which forecasts the corresponding production rate (oil, gas or water) at the \( k \)th sampling instant, using \( p \) previous values of such rate, \( p \) previous values of the opening of the valve in all injecting segments (input) and \( p \) previous values of the innovation (estimation) error of the neural network.

**Control stage** Using the available Kalman model, a predictive control strategy is designed according to step 4 of the methodology for subcool control, to find the daily valve opening in every segment of the injection well during that year, in order to reach the oil production set-point corresponding to that year.

**Case studies**

Two synthetic reservoir models were created, one based on an idealized heterogeneity pattern (control scenario) and the other on logs from the Orinoco Belt region (optimization scenario). General reservoir and grid data, injection and production parameters and other properties are identical for both case studies and are specified in Fig. 2 and Tables 1 - 3. Specifically the grid is composed of \( 20 \times 65 \times 15 \) blocks in the x-, y-, and z-directions, respectively. The vertical spacing between the injector and producer well is 24.6ft with 10 segments (completions) each, with both wells of 3280ft length. The reservoir has an initial pressure of 384.9 psi, an initial oil and water saturation of 0.85 and 0.15, respectively, and an initial temperature of 136.4°F. Both the overburden and underburden of the reservoir model are assumed to have a temperature of 136.4°F, a heat capacity of 17.4 BTU/ft\(^3\)*°F and a thermal conductivity of 0.503 BTU/ft*h*°F.

![Figure 2—Illustration of the grid (case study).](image-url)
The reservoir model is associated with an idealized heterogeneity pattern consisting of constant values of 0.33 for porosity and 3400 and 680 md for horizontal and vertical permeability, respectively. There are ten controlled variables (subcool in each segment of the producing well), and ten input variables (valve opening in each segment of the injecting well). The proposed control strategy is compared with a

Table 1—Reservoir model parameters (case study).

<table>
<thead>
<tr>
<th>Reservoir data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock compressibility [psi⁻¹]</td>
<td>6.62×10⁻⁵</td>
</tr>
<tr>
<td>Rock heat capacity [BTU/ft³°F]</td>
<td>35</td>
</tr>
<tr>
<td>Rock thermal conductivity [BTU/ft³°F]</td>
<td>0.838</td>
</tr>
<tr>
<td>Reservoir initial temperature [°F]</td>
<td>136.4</td>
</tr>
<tr>
<td>Reservoir initial pressure [psi]</td>
<td>384.9</td>
</tr>
<tr>
<td>Initial Sₚ</td>
<td>0.15</td>
</tr>
<tr>
<td>Initial Sₚ</td>
<td>0.85</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Injection and production parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wells vertical spacing [ft]</td>
<td>24.6</td>
</tr>
<tr>
<td>Injection pressure [psi]</td>
<td>435.1</td>
</tr>
<tr>
<td>Production pressure [psi]</td>
<td>81.9</td>
</tr>
<tr>
<td>Number of segments</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Oil properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal capacity [BTU/lb*°F]</td>
<td>0.407</td>
</tr>
<tr>
<td>Thermal expansion [°F⁻¹]</td>
<td>4.64×10⁻⁴</td>
</tr>
<tr>
<td>Density [lb/ft³]</td>
<td>62.18</td>
</tr>
<tr>
<td>Compressibility [psi⁻¹]</td>
<td>3.88×10⁻⁶</td>
</tr>
</tbody>
</table>

Table 2—Reservoir and grid data (case study).

<table>
<thead>
<tr>
<th>Gridblocks</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>65</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Gridblock size [ft]</td>
<td>164</td>
<td>4.92</td>
<td>4.92</td>
</tr>
<tr>
<td>Reservoir size [ft]</td>
<td>3280</td>
<td>319.8</td>
<td>73.8</td>
</tr>
</tbody>
</table>

Table 3—Relative permeability data (case study).

<table>
<thead>
<tr>
<th>Oil-Water System</th>
<th>Gas-Oil System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sw</td>
</tr>
<tr>
<td>-----------------</td>
<td>-----</td>
</tr>
<tr>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>0.30</td>
<td>0.01</td>
</tr>
<tr>
<td>0.35</td>
<td>0.01</td>
</tr>
<tr>
<td>0.40</td>
<td>0.03</td>
</tr>
<tr>
<td>0.45</td>
<td>0.04</td>
</tr>
<tr>
<td>0.50</td>
<td>0.07</td>
</tr>
<tr>
<td>0.55</td>
<td>0.10</td>
</tr>
<tr>
<td>0.60</td>
<td>0.15</td>
</tr>
<tr>
<td>0.65</td>
<td>0.20</td>
</tr>
<tr>
<td>0.70</td>
<td>0.27</td>
</tr>
<tr>
<td>0.75</td>
<td>0.35</td>
</tr>
<tr>
<td>0.80</td>
<td>0.45</td>
</tr>
<tr>
<td>0.85</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

Subcool control
The reservoir model is associated with an idealized heterogeneity pattern consisting of constant values of 0.33 for porosity and 3400 and 680 md for horizontal and vertical permeability, respectively. There are ten controlled variables (subcool in each segment of the producing well), and ten input variables (valve opening in each segment of the injecting well). The proposed control strategy is compared with a
decentralized PID strategy, i.e., ten PID controllers are implemented, one for each injecting-producing segment pair. After an initial heating period, both control strategies are implemented for a period of one year (365 days), with a sampling time of one (1) day and a subcool set-point of 5°F. The parameters for each controller of the proposed strategy were $p = 2, N_p = N_c = 4, Q = I_4$ and $R = I_4$. On the other hand, the parameters of each PID controller were $K = -0.5$, $T_i = 1000$ and $T_d = 0$.

**NPV optimization under uncertainty**

The petrophysical properties specified in the reservoir models are based on logs from Orinoco Belt region, more specifically, the O-12 operational unit located in the Carabobo block and were generated using the Sequential Gaussian Simulation method with the continuity models described in Table 4 where the direction of maximum continuity (greater range) coincides with the well direction. Basic statistical properties for the porosity and permeability fields are shown in Table 5 and to ensure correlation between porosity and permeability, the latter is simulated using Cokriging with a correlation coefficient of 0.9. Furthermore in order to account for geological uncertainty, fifteen (15) reservoir model realizations were generated with risk aversion modeled using $\lambda$ equal to 0, 1, 2 and 3 in objective function (33). For a neutral risk aversion scenario ($\lambda$ equal to 0), the results of the proposed optimization strategy are compared with those corresponding to a maximum permissible injection rate (3361 STB/D; CI-Max) and the expected value of four constant steam injection rates (840, 1680, 2521 and 3361 STB/D; CI-Avg). In all instances, the production horizon was set to be four (4) years with the economic parameters shown in Table 6.

**Table 4—Variogram parameters used in the geostatistical modeling of porosity and permeability (case study).**

<table>
<thead>
<tr>
<th>Continuity model</th>
<th>Greater (ft)</th>
<th>Smaller (ft)</th>
<th>Vertical (ft)</th>
<th>Nugget</th>
<th>Covariance model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>1640</td>
<td>656</td>
<td>49.2</td>
<td>0</td>
<td>Spherical</td>
</tr>
<tr>
<td>Permeability</td>
<td>2788</td>
<td>1394</td>
<td>49.2</td>
<td>0</td>
<td>Spherical</td>
</tr>
</tbody>
</table>

**Table 5—Parameters for the Conditional Sequential Gaussian Simulation of porosity and permeability fields (case study).**

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Maximum</th>
<th>Expected Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity (%)</td>
<td>22</td>
<td>35</td>
<td>25</td>
<td>6</td>
</tr>
<tr>
<td>Permeability (darcy)</td>
<td>6</td>
<td>1.9</td>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 6—Economic parameters values (case study).**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_o$ [USD/STB]</td>
<td>40</td>
</tr>
<tr>
<td>$p_g$ [USD/MSCF]</td>
<td>4</td>
</tr>
<tr>
<td>$c_w$ [USD/STB]</td>
<td>5</td>
</tr>
<tr>
<td>$c_s$ [USD/STB]</td>
<td>10</td>
</tr>
<tr>
<td>$\Delta t_i$ [days]</td>
<td>1</td>
</tr>
<tr>
<td>$r$ [%]</td>
<td>34</td>
</tr>
<tr>
<td>$\beta$ [%]</td>
<td>15</td>
</tr>
</tbody>
</table>

**Performance measures**

This section presents the evaluation criteria to assess the performance of the proposed approach for both subcool control and NPV optimization under uncertainty.

**Subcool control**

In this context, the performance measures are:

a. Mean square error (MSE): calculated as,
b. Reaching time \((t_r)\): time necessary for the subcool to be within 1°F of its set-point for the first time, and represents a measure of the speed of response.

c. Settling time \((t_s)\): time necessary for the subcool to enter and remain in an interval of ±1°F around its set-point and represents a measure of the stabilization time.

d. Mean value \((MV)\) and standard deviation \((STD)\) of the subcool after \(t_s\): the former measures the value around which the subcool stabilizes (after settling time), while the latter measures the variability around the mean.

e. Energy spent in the control signal \((CE)\): measures the energy required for performing the control task and is calculated as:

\[
CE = \sum_{k=1}^{K} u(k)^2
\]

NPV optimization under uncertainty

The results of the proposed NPV optimization approach are evaluated against constant injection strategies using the following performance measures:

a. Mean of the NPV and the contribution of individual terms, such as, income from oil and gas production, and costs associated to produced water handling and steam injection.

b. Steam-oil ratio \((SOR)\): defined as the quotient between the volume of cold water equivalent (to the volume of steam injected) and the volume of oil produced, i.e.

\[
SOR = \frac{\text{Volume of cold water equivalent injected}}{\text{Volume of oil produced}}
\]

with lower values of SOR suggesting more efficient recovery processes.

Results and Discussion

Subcool control

Fig. 3 shows the subcool profiles for segments representative (C1, C5 and C10) of the producing well obtained using the proposed MPC approach (Fig. 3a) and PID (Fig. 3b). Note the significantly smoother subcool profiles exhibited by the proposed approach compared with those corresponding to PID control.
In addition, Table 7 shows the values of the performance measures for both control strategies for the above-referenced segments. Specifically, with respect to PID the MPC approach exhibited:

- Slower response, as indicated by higher MSE values ([1.40–2.28] vs. [0.62–1.07]) and larger \(t_r\) ([10–28] days vs. [3–5] days). This is not a critical issue, i.e., it should not have a meaningful impact on oil recovery, considering that this affects the performance during a relatively short period of time early in the control process (transient period).

- Better performance in steady-state, due to faster stabilization as suggested by smaller \(t_s\) values ([31–44] days vs. [20–148] days), mean value of measured subcool significantly closer to the set-point ([4.97–4.99] °F vs. [4.41–4.58] °F), and slightly lower standard deviation values ([0.3–0.6] vs. [0.45–0.51]). This is a key aspect because it translates in a better long-term performance of the SAGD process.

Figure 3—Subcool control: measured subcool and set-point for segments 1, 5 and 10 of the producing well, for (a) the proposed strategy (MPC) and (b) PID.

<table>
<thead>
<tr>
<th></th>
<th>MPC</th>
<th>PID</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C5</td>
</tr>
<tr>
<td>MSE</td>
<td>2.28</td>
<td>1.40</td>
</tr>
<tr>
<td>(t_r) [days]</td>
<td>10</td>
<td>32</td>
</tr>
<tr>
<td>(t_s) [days]</td>
<td>31</td>
<td>32</td>
</tr>
<tr>
<td>(MV \pm STD) [°F]</td>
<td>4.97±0.43</td>
<td>4.95±0.3</td>
</tr>
<tr>
<td>CE</td>
<td>15.32</td>
<td>9.22</td>
</tr>
</tbody>
</table>
• Lower control energy spending ([9.22–15.32] vs. [30.17–39.9]). This can be also seen from significantly smoother control signals offered by the proposed strategy (Fig. 4a) compared with the signals corresponding to PID (Fig. 4b). This should be expected because the objective function minimized in the design of the predictive control strategy includes a term that penalizes the control energy expenditure, as opposed to the PID, in which such a design criteria cannot be explicitly considered.

NPV optimization under uncertainty

Table 8 shows the results of the proposed approach, significantly outperforming those associated with constant injection rate scenarios. Specifically, with respect to CI-Max and CI-Avg, the proposed approach leads to:

- Greatest NPV mean, 27% and 387% larger, respectively
- Slightly lower oil and similar gas production volumes, the former 9% and 3% lower, respectively
- Lower steam injection, and water production volumes, the former 24% and 60% less, and the latter 54% and 25% less, respectively.
- Significantly lower SOR, approximately 30% and 50% less, respectively

Table 8—NPV optimization: results of the proposed approach (MPC) and constant steam injection strategies (CI-Max and CI-Avg), considering a neutral risk aversion scenario.

<table>
<thead>
<tr>
<th></th>
<th>Mean NPV</th>
<th>Mean income due to oil sales</th>
<th>Mean income due to gas sales</th>
<th>Mean cost of steam injection</th>
<th>Mean cost of handling produced water</th>
<th>SOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>USD x 10^7</td>
<td>USD x 10^7</td>
<td>USD x 10^8</td>
<td>USD x 10^6</td>
<td>USD x 10^8</td>
<td></td>
</tr>
<tr>
<td>MPC</td>
<td>1.59</td>
<td>2.84</td>
<td>4.98</td>
<td>4.40</td>
<td>8.2</td>
<td>3.66</td>
</tr>
<tr>
<td>CI-Max</td>
<td>0.41</td>
<td>3.12</td>
<td>5.08</td>
<td>9.28</td>
<td>17.9</td>
<td>12.25</td>
</tr>
<tr>
<td>CI-Avg</td>
<td>1.25</td>
<td>2.93</td>
<td>4.90</td>
<td>5.80</td>
<td>11</td>
<td>7.21</td>
</tr>
</tbody>
</table>
Table 9 shows the results of NPV optimization under uncertainty when modeling increasingly higher risk aversion factors, i.e., $\lambda$ of 0, 1, 2 and 3, namely, objective function ($F$) value, corresponding NPV mean ($\mu$) and standard deviation ($\sigma$), and confidence level interpretation. For instance, for $\lambda=0$ the confidence level can be interpreted as 50%, so it is expected a NPV greater than $15.95 \times 10^6$ USD with a 0.5 probability. Note that higher values of $\lambda$ translates into a more conservative estimate of NPV (e.g., $\lambda=3$, $10.55 \times 10^6$ USD).

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$F$ USD $\times 10^6$</th>
<th>$\mu$ USD $\times 10^6$</th>
<th>$\sigma$ USD $\times 10^6$</th>
<th>Confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15.93</td>
<td>15.93</td>
<td>2.86</td>
<td>50%</td>
</tr>
<tr>
<td>1</td>
<td>14.64</td>
<td>16.96</td>
<td>2.32</td>
<td>84.1%</td>
</tr>
<tr>
<td>2</td>
<td>12.23</td>
<td>16.62</td>
<td>2.2</td>
<td>97.7%</td>
</tr>
<tr>
<td>3</td>
<td>10.55</td>
<td>16.83</td>
<td>2.1</td>
<td>99.8%</td>
</tr>
</tbody>
</table>

Conclusions and recommendations

This paper presented a computationally efficient nonlinear model-based adaptive-predictive control strategy as an alternative to PID, for subcool control and NPV optimization of SAGD processes. Some advantages of the proposed strategy: (i) the complex reservoir dynamics is described using a nonlinear model, resulting in a more accurate approximation, (ii) accounts for geological uncertainty, (iii) the design is based on an optimal time-varying local linear equivalent of the nonlinear models, which yields a closed solution of the predictive control optimization problem, (iv) it handles MIMO processes and operational constraints in a straightforward manner and (v) since a Kalman model is used to estimate the state in the calculation of the control input, a good performance may be obtained in the presence of model uncertainty and measurement noise.

The proposed strategy was compared with a decentralized PID for subcool control, in terms of response speed (mean square error and reaching time), steady state behavior (settling time, measured subcool mean and standard deviation) and control energy spending. While the proposed approach offered a slower response (not a critical issue in terms of oil recovery), it significantly outperformed the PID control during steady state and in control energy spending. On the other hand, the effectiveness for NPV optimization under uncertainty was demonstrated against constant steam injection strategies (CI-Max and CI-Avg) considering NPV mean, steam injection, water produced, and SOR, and when modeling alternative risk aversion scenarios.

Future work will evaluate the proposed approach for subcool control and NPV optimization under uncertainty in more general heterogeneous reservoirs and potential applications in the optimization of other enhanced oil recovery processes (e.g. polymer flooding, CO2 flooding).

References


